NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES

SCHOOL OF NATURAL AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

QUALIFICATION: Bachelor of Science; Bachelor of Science in Applied Mathematics and Statistics			
QUALIFICATION CODE:	07BSOC; 07BSAM	**LEVEL:**	5
COURSE CODE:	LIA502S	**COURSE CODE:**	LINEAR ALGEBRA 1
SESSION:	JUNE 2023	**PAPER:**	THEORY
DURATION:	3 HOURS	**MARKS:**	100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER:	DR. DSI IIYAMBO
MODERATOR:	DR. N CHERE

INSTRUCTIONS
1. Attempt all the questions in the booklet provided. 2. Show clearly all the steps used in the calculations. 3. All written work must be done in black or blue inked, and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

Question 1

Consider the vectors $\mathbf{p} = 3\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$, $\mathbf{q} = \mathbf{i} - 3\mathbf{j} + 12\mathbf{k}$ and $\mathbf{r} = \mathbf{i} - 6\mathbf{k}$

a) Find a vector of magnitude 3 in the direction of $\mathbf{q}$. [6]

b) Find the angle (*in degrees*) between $\mathbf{p}$ and $\mathbf{r}$. Give you answer correct to 1 d.p. [8]

c) Calculate the projection of $\mathbf{p}$ onto $\mathbf{r}$, $\text{Proj}_{\mathbf{r}}\mathbf{p}$. [5]

Question 2

Consider the matrices $A = \begin{pmatrix} -1 & 0 & 2 \\ 3 & 4 & 5 \\ 1 & 2 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 \\ 3 & 2 \\ 3 & -2 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$.

a) Without evaluating the whole product, determine the elements

(i) in the third row and second column of AB [3]

(ii) in the second row and second column of BC [3]

b) Given that $\alpha\, tr(A) + 10\, tr(C) = 12$, find the value(s) of α which satisfies this equation. [4]

Question 3

Let $F = \begin{pmatrix} 3 & 5 & x \\ y & 8 & 4 \\ -3 & z & 3 \end{pmatrix}$.

a) Given that the matrix F is symmetric, give the values of x, y and z. [5]

b) Prove that if A and B are both $n \times n$ symmetric matrices such that $AB = BA$, then AB is a symmetric matrix. [6]

c) Prove that if A is an invertible symmetric matrix, then A^{-1} is also symmetric. [6]

Question 4

Consider the matrix $A = \begin{pmatrix} -1 & 1 & 2 \\ 3 & 0 & -5 \\ 1 & 7 & 2 \end{pmatrix}$.

a) Use the *Cofactor expansion method*, expanding along the second column, to evaluate the determinant of A. [6]

b) Is A invertible? If it is, *use the adjoint method* to find A^{-1}. [14]

c) Find $\det\left(3(2A)^{-1}\right)$. [6]

Question 5

Use the *Gaussian elimination method* to find the solution of the following system of linear equations, if it exists.

$$\begin{aligned} x + 2y &= 2 \\ 2x + z &= 1 \\ 3x + 2y + z &= 3 \end{aligned}$$

[8]

Question 6

a) Prove that in a vector space, the negative of a vector is unique. [9]

b) Let M_{nn} be a vector space whose elements are all the $n \times n$ matrices, with the usual addition and scalar multiplication for matrices. Determine whether the following set is a subspace of M_{nn}.

$$S = \{A \in M_{nn} \mid tr(A) = 0\}$$

[11]
